

Using Operations Research to Determine the Optimal Three Mashers for Three Stages of Broiler Feeding

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Abstract. In this research, one of the methods of operations research, a linear programming method, was used to build a mathematical model to minimize the costs of feeding chickens for the production of white meat. The feeding system adopted for the first three periods from the age of one day to the age of 22 days. The second period starts from age 23 days to age 42 days and the third period starts from the age of 43 days until the stage of direct marketing by taking advantage of feed materials available in local markets or imported from outside Iraq to choose three economic diets, and each diet is suitable for feeding the chickens for the production of white meat. These three stages of nutrition should ensure the presence of all the essential elements to be provided in chicken diets such as proteins, energy represented, fats, fiber, vitamins, mineral salts, amino acids and other elements required to be available in the poultry food at the same rates determined by the dietitian and for each three phase of feeding. The use of these feeds extracted through the application of one of the methods of operations research will lead to minimizing the cost of poultry feeding as well as reducing the time required for the arrival of chickens to the stage of marketing and profit taking.

Keywords: linear programming, broiler feeding, broiler diet, feed formulation

1. Introduction

Linear programming method, are effective methods that have achieved wide success in various areas of life. I will mention some of the production planning, transportation problems, distribution of energy sources, financial planning, health care planning, hospital management and other areas. One of the methods of operations research in the field of poultry nutrition through the identification of optimal feed mixes for the stages of breeding (the beginning stage - the growth stage - the end stage) and to take advantage of the feed materials available in the local markets for the production of chicken feed rations that meet the needs basic and necessary for the required chicken feed at the lowest possible financial costs, and this will increase the productivity of the profitability of these fields.

Increasing the profitability of poultry farms is achieved primarily through minimizing the cost of feeding poultry during the three feeding periods. Feeding chickens has become one of the most important challenges facing the owners of poultry farms. Feeding chickens to a minimum while providing all the basic needs required for feeding poultry without increasing or decreasing to obtain the highest productivity performance in the production of white meat, in addition to reducing the other expenses required by the process of raising chickens dedicated to produce meat whenever possible. Poultry farming in Iraq is facing the problem of the lack of use of modern scientific methods in



feeding poultry and reliance on the personal experiences of some owners of poultry fields, which led to high production costs and low productivity profitability of these fields, which led many owners of poultry fields to close their fields for lack of profits.

Therefore, some research needed to reduce the cost of feeding, which form a large percentage of the cost of breeding chickens by applying one of the methods of operations research.

2. Materials And Methods

N R C (1994) is the main source of the data in the tables 2, 3, 4

Table 1. Upper limits and the prices of the broiler diet ingredient

N.	The ingredient	Upper limit of Ingredient in 100 kgm of diet	Price of 1 kgm		The ingredient	Upper limit of ingredient in 100 kgm of diet	Price of 1 kgm
1	The wheat	70	510	14	Fish meal	5	1750
2	The barley	20	430	15	Bone meal	2.5	850
3	The maize	70	490	16	The broad bean	10	510
4	White corn	10	410	17	The white Beans	10	1210
5	The Rice	70	290	18	The Chickpeas	10	710
6	The coarse wheat bran	5	400	19	The Lentils	10	710
7	The fine wheat bran	5	400	20	Dehydrated alfalfa 17% protein	7	610
8	maiz gluten 42% protein	5	958	21	Dehydrated alfalfa leaves	7	610
9	maize gluten 60% protein	5	1450	22	Limestone	0.5	110
10	Soybean meal 44% protein	40	838	23	Vegetable oils	4	1810
11	Soybean meal 49% protein	30	910	24	Crude vegetable oils	4	1500
12	Sunflower meal	40	460	25	Vegetable protein concentrates	10	1930
13	The Sesame meal	3	3000	26	Animal protein concentrates	10	1800
				27	Sodium chloride	0.25	110

Table 2. Chemical analysis of broiler diet ingredient

N.	The Ingredient	Energy	Lysine%	Methionine%	Phosphor%	Calcium%	Crude Protein%	Sodium%
1	the wheat	3250	0.39	0.37	0.12	0.05	13	0.07
2	the barely	2750	0.39	0.37	0.16	0.08	9	0.02
3	the maize	3370	0.24	0.4	0.11	0.07	8.72	0.01
4	white corn	3210	0.25	0.35	0.13	0.04	11.4	0.01
5	The rice	3150	0.3	0.3	0.04	0.09	8.18	0.11
6	The coarse wheat bran	1250	0.53	0.42	0.34	0.16	14.1	0.3
7	The fine wheat bran	1650	0.59	0.47	0.37	0.15	15.5	0.07
8	maiz gluten 42% protein	3335	0.73	1.91	0.15	0.45	42	0.1
9	maiz gluten 60% protein	3750	1.29	2.79	0.19	0	60	0.01
10	soybean meal 44% protein	2300	2.91	1.33	0.29	0.32	45	0.24
11	soybean meal 49% protein	2500	3.17	1.47	0.19	0.26	50	0.34
12	sunflower meal	1960	1.73	2.22	0.16	0.38	46.9	
13	The sesame meal	2415	1.09	1.86	0.42	2.02	44.5	0.3
14	The fish meal	2963	4.83	2.32	2.95	5.02	64.7	
15	bone meal	1398	0.87	0.29	14	30	12.1	0.46
16	The broad bean	2520	1.55	0.47	0.12	0.17	26.1	0.08
17	The white beans	2330	1.1	0.48	0.15	0.13	24	
18	The chickpeas	2756	1.34	0.59	0.18	0.2	20.8	
19	The lentils	2647	1.73	0.41	0.11	0.52	23.5	
20	Dehydrated alfalfa 17% protein	1453	0.82	0.51	0.24	1.3	17.3	0.18
21	Dehydrated alfalfa leaves 20% protein	1580	0.9	0.55	0.27	1.5	21.1	0.19
22	The lime stone				30	38		
23	Crude vegetable oils	7010						
24	Vegetable oils	8900						

25	Vegetable protein concentrates						52	
26	Animal protein concentrates						50	
27	sodium chloride							50

Source NRC (1994)

Table 3. Percentage of humidity, fiber, lipids and vitamins amounts in broiler diet ingredient

N.	The Ingredient	Thiamin mg/kgm	Niacin mg/kgm	Pantothenic mg/kgm	Riboflavin mg/kgm	Crude Fiber %	Lipids %	Ash %	Humidity %
1	the wheat	4.4	56.6	12.1	1.2	2.4	1.9	1.6	11
2	the barely	5.1	57.4	6.5	2	5	1.9	2.4	11
3	the maize	4	22.9	5	1.1	2	3.9	1.8	12
4	white corn	3.9	42.7	11.4	1.2	2	2.8	1.7	11
5	The rice	0.6	14.1	3.3	0.6	9	1.9	4.5	11
6	The coarse wheat bran	7.9	209	29	3.1	10	4.1	6.1	11
7	The fine wheat bran	2.6	20	13	0.9	3	2.9	2.1	11
8	maiz gluten 42% protein	0.2	49.9	10.3	1.5	4	2.3	2.4	9
9	maiz gluten 60% protein	0.2	5.5	2.9	2.2	1.3	1.7	2	9
10	soybean meal 44% protein	4	30.4	15.2	2.6	6	4.7	5.7	10.4
11	soybean meal 49% protein	6.5	26.8	14.5	3.3	3	0.8	5.6	10
12	sunflower meal		220	10.1	3.3	5	5.1	9.3	7
13	The sesame meal	2.86	30.8	6.38	3.74	14	2.8	7.1	10
14	The fish meal	0.2	47.1	3.3	4.6	1	4.4	21.7	8
15	bone meal	3.6	10.6	33.9	18.9	2	3.2	71.8	5
16	The broad bean					5.7	1.5	3.6	13.4
17	The white beans								
18	The chickpeas								
19	The lentil		500	82.9	44.4				
20	Dehydrated alfalfa 17% protein	4	54.6	32.8	15.4	24.8	2.6	9	7

21	Dehydrated alfalfa leaves	0.9	46	39.4	2.4	20.1	3.6	10.3	6.9
22	The lime stone								
23	Vegetables oils						90		
24	Crude vegetable oils						90		
25	Vegetable protein concentrates								
26	Animal protein concentrates								
27	Sodium chloride								

Source NRC (1994)

Source NRC (1994)

Table 4. Requirement of the broiler diet in the three intervals of nutrition

N.	The Ingredient	Initial Mash	Grower Mash	Finisher mash
1	Crude protein %	24	22	20
2	Energy kilocalorie/kgm	2800	3000	3300
3	Crude Fiber %	3.5	4	4
4	Crude Fat %	5	6	7
5	Energy/Protein	117	136	165
6	Ash %	0.50%	0.50%	0.50%
7	Calcium %	1	0.9	0.8
8	Sodium %	0.2	0.15	0.12
9	Chloride %	0.2	0.15	0.12
10	Phosphor %	0.45	0.35	0.3
11	Lysine %	1.1	1	0.85
12	Methionine %	0.5	0.38	0.32
13	Methionine + Systine	0.9	0.72	0.61
14	Riboflavin (mg)	3.6	3.5	3
15	Niacin (mg)	35	30	25
16	Pantothenic (mg)	10	10	10
17	Niacin (mg)	35	30	25
18	Folic Acid (mg)	0.55	0.55	0.5
19	Thiamin (mg)	1.8	1.7	1.6

Source NRC (1994)

3. The Mathematical Model

3.1. Decision Variables

Table 5. Decision variables for the three mashes

N.	Ingredient name	Symbol of the decision variable in the Initial mash	Symbol of the decision variable in the grower mash	Symbol of the decision variable in the finisher mash
1	The wheat	A1	B1	C1
2	The barley	A2	B2	C2
3	The maize	A3	B3	C3
4	The white corn	A4	B4	C4
5	The Rice	A5	B5	C5
6	The coarse wheat bran	A6	B6	C6
7	The fine wheat bran	A7	B7	C7
8	Maize gluten 42% protein	A8	B8	C8
9	Maize gluten 60% protein	A9	B9	C9
10	Soybean meal 44% protein	A10	B10	C10
11	Soybean meal 49% protein	A11	B11	C11
12	Sunflower meal	A12	B12	C12
13	The Sesame meal	A13	B13	C13
14	Fish meal	A14	B14	C14
15	Bone meal	A15	B15	C15
16	The bean	A16	B16	C16
17	The white Beans	A17	B17	C17
18	The Chickpeas	A18	B18	C18
19	The Lentil	A19	B19	C19
20	Dehydrated alfalfa 17% protein	A20	B20	C20
21	Dehydrated alfalfa leaves 20%protein	A21	B21	C21
22	Limestone	A22	B22	C22
23	Vegetable oils	A23	B23	C23
24	Crude vegetable oils	A24	B24	C24
25	Vegetable protein concentrates	A25	B25	C25
26	Animal protein concentrates	A26	B26	C26
27	Sodium chloride	A27	B27	C27

3.2. *The Initial Mash*

3.3. *The objective function*

Minimize the summation of the cost of 100 kgm. of the initial mash

$$\bullet \text{Min } z = 510A1 + 430A2 + 490A3 + 410A4 + 290A5 + 400A6 + 400A7 + 958A8 + 1450A9 + 838A10 + 910A11 + 460A12 + 3100A13 + 1750A14 + 850A15 + 510A16 + 1210A17 + 710A18 + 710A19 + 610A20 + 610A21 + 110A22 + 110A23 + 1810A24 + 1930A25 + 1750A26 + 110A27 \dots \dots \dots (1).$$

3.4. *The Constraints*

- *Constraint of the wheat in the initial mash: $A1 \leq 70$ (2).*
- *Constraint of the barely in the initial mash: $A2 \leq 20$ (3).*
- *Constraint of the maiz in the initial mash: $A3 \leq 70$ (4).*
- *Constraint of the white corn in the initial mash: $A4 \leq 10$ (5).*
- *Constraint of the rice in the initial mash: $A5 \leq 70$ (6).*
- *Constraint of the coarse wheat brane in the initial mash: $A6 \leq 5$ (7).*
- *Constraint of the fine wheat brane in the initial mash: $A7 \leq 5$ (8).*
- *Constraint of the maize gluten 42% protein in the initial mash: $A8 \leq 5$ (9).*
- *Constraint of the maize gluten 60% protein in the initial mash: $A9 \leq 5$ (10).*
- *Constraint of the soybean meal 44% protein in the initial mash: $A10 \leq 40$ (11).*
- *Constraint of soybean meal 49% protein in the initial mash: $A11 \leq 30$ (12).*
- *Constraint of sunflower meal in the initial mash: $A12 \leq 40$ (13).*
- *Constraint of the of the sesame meal in the initial mash: $A13 \leq 3$ (14).*
- *Constraint of the fish meal in the initial mash: $A14 \leq 5$ (15).*
- *Constraint of the bone meal in the initial mash: $A15 \leq 2.5$ (16).*
- *Constraint of the bean in the initial mash: $A16 \leq 10$ (17).*
- *Constraint of the white beans in the initial mash: $A17 \leq 10$ (18).*
- *Constraint of the chick peas in the initial mash: $A18 \leq 10$ (19).*
- *Constraint of the lentils in the initial mash: $A19 \leq 10$ (20).*
- *Constraint of the dehydrated alfalfa 17% protein in the initial mash: $A20 \leq 7$ (21).*
- *Constraint of the dehydrated alfalfa leaves 20% protein in the initial mash: $A21 < 7$ (22).*
- *Constraint of the Lime stone in the initial mash: $A22 \leq 1$ (23).*
- *Constraint of the vegetable oils in the initial mash: $A23 \leq 4$ (24).*
- *Constraint of the crude vegetable oils in the initial mash: $A24 \leq 4$ (25).*
- *Constraint of the vegetable protein concentrates & vitamins in the initial mash: $A25 \leq 10$.. (26).*
- *Constraint of the animal protein concentrates & vitamins in the initial mash: $A26 \leq 1$ (27).*
- *Constraint of the sodium chloride in the initial mash: $A27 \leq 0.25$ (28).*

Constraint of all ingredients in the initial mash:

$$\bullet A1 + A2 + A3 + A4 + A5 + A6 + A7 + A8 + A9 + A10 + A11 + A12 + A13 + A15 + A16 + A17 + A18 + A19 + A20 + A21 + A22 + A23 + A24 + A25 + A26 + A27 = 100 \dots \dots \dots (29)$$

- *Constraint of the lower limit of the legumes in the initial mash: $A16+A17+A18+A19 \geq 3$ (30)*
- *Constraint of the upper limit for the legumes in the initial mash: $A16+A17+A18+A19 \leq 10$ (31)*
- *Constraint of the meals in the initial mash: $A8+A9+A10+A11+A12+A13 \leq 40$ (32)*
- *Constraint of the grains and another energy sources in the initial mash: $A1+ A2+ A3+ A4+ A5+ A6+ A7 \leq 50$ (33)*
- *Constraint of the oils in the initial mash: $A23+A24 \leq 4$ (34)*
- *Constraint of the concentrated proteins in the initial mash: $A25+A26 \leq 10$ (35)*

Constraint of the percentage in the initial mash:

- *$13A1+ 9A2+ 8.72A3+ 11.4A4+ 8.18A5+ 14.1A6+ 15.5A7+ 42A8+ 60A9+ 45A10+ 50A11+ 46.9A12+ 44.5A13+ 64.7A14+ 12.1A15+ 26.1A16+ 24A17+ 20.8A18+ 23.5A19+ 17.3A20+ 21.1A21+ 50A25+ 50A26 = 2400$ (36)*
- *Constraints of the metabolizable energy in the initial mash:*
- *$3250A1+ 2750A2+ 3370A3+ 3210A4+ 3150A5+ 1250A6+ 1650A7+ 3335A8+ 3760A9+ 2300A10+ 2500A11+ 1960A12+ 2416A13+ 2963A14+ 1398A15+ 2520A16+ 2330A17+ 2756A18+ 2647A19+ 1453A20+ 1580A21+ 7010A23+ 8900A24 = 280000$ (37)*

Constraint the upper limit of the lipid percentage in the initial mash:

- *$1.9A1+ 1.9A2+ 3.9A3+ 2.8A4+ 1.9A5+ 4.1A6+ 2.94A7+ 2.3A8+ 1.7A9+ 4.7A10+ 0.8A11+ 5.1A12+ 2.8A13+ 4.4A14+ 3.2A15+ 1.5A16+ 2.6A20+ 3.6A21+ 90A23+ 90A24 \leq 700$ (38)*

Constraint the lower limit of the lipid percentage in the initial mash:

- *$1.9A1+ 1.9A2+ 3.9A3+ 2.8A4+ 1.9A5+ 4.1A6+ 2.94A7+ 2.3A8+ 1.7A9+ 4.7A10+ 0.8A11+ 5.1A12+ 2.8A13+ 4.4A14+ 3.2A15+ 1.5A16+ 2.6A20+ 3.6A21+ 90A23+ 90A24 \geq 400$ (39)*

Constraint of the Lower limit of the humidity percentage in the initial mash:

- *$11A1+ 11A2+ 12A3+ 11A4+ 11A5+ 11A6+ 11A7+ 9A8+ 9A9+ 10.4A10+ 10A11+ 7A12+ 10A13+ 8A14+ 5A15+ 13.4A16+ 7A20+ 10.3A21 \geq 500$ (40)*

Constraint of the upper limit of the humidity percentage in the initial mash:

- *$11A1+ 11A2+ 12A3+ 11A4+ 11A5+ 11A6+ 11A7+ 9A8+ 9A9+ 10.4A10+ 10A11+ 7A12+ 10A13+ 8A14+ 5A15+ 13.4A16+ 7A20+ 10.3A21 \leq 1000$ (41)*

Constraint of the lower limit of the crude fiber percentage in the initial mash:

- *$2.4A1+ 5A2+ 2A3+ 2A4+ 9A5+ 10A6+ 3A7+ 4A8+ 1.3A9+ 6A10+ 3A11+ 5A12+ 14A13+ A14+ 2A15+ 5.7A16+ 24.8A20+ 20.1A21 \geq 300$ (42)*

Constraint of the upper limit of the crude fiber percentage in the initial mash:

• $2.4A1+ 5A2+ 2A3+ 2A4+ 9A5+ 10A6+ 3A7+ 4A8+ 1.3A9+ 6A10+ 3A11+ 5A12+ 14A13+ A14+ 2A15+ 5.7A16+ 24.8A20 +20.1A21 \leq 700 \dots \dots \dots (43)$

Constraint of the lower limit of the ash percentage in the initial mash:

• $1.6A1+ 2.4A2+ 1.8A3+ 1.7A4+ 4.5A5+ 6.1A6+ 2.1A7+ 2.4A8+ 2A9+ 5.7A10+ 5.6A11+ 9.3A12+ 7.1A13+ 21.7A14+ 71.8A15+ 3.6A16+ 9A20+ 10.3A21 \geq 200 \dots \dots \dots (44)$

Constraint of the upper limit of the ash percentage in the initial mash:

• $1.6A1+ 2.4A2+ 1.8A3+ 1.7A4+ 4.5A5+ 6.1A6+ 2.1A7+ 2.4A8+ 2A9+ 5.7A10+ 5.6A11+ 9.3A12+ 7.1A13+ 21.7A14+ 71.8A15+ 3.6A16+ 9A20+ 10.3A21 \leq 500 \dots \dots \dots (45)$

Constraint of the Lower limit of phosphor in the initial mash:

• $0.12A1+ 0.16A2+ 0.11A3+ 0.13A4+ 0.04A5+ 0.34A6+ 0.37A7+ 0.15A8+ 0.19A9+ 0.29A10+ 0.19A11+ 0.16A12+ 0.42A13+ 2.95A14+ 14A15+ 0.12A16+ 0.15A17+ 0.18A18+ 0.11A19+ 0.24A20+ 0.27A21 \geq 39 \dots \dots \dots (46)$

Constraint of the upper limit of the phosphor percentage in the initial mash:

• $0.12A1+ 0.16A2+ 0.11A3+ 0.13A4+ 0.04A5+ 0.34A6+ 0.37A7+ 0.15A8+ 0.19A9+ 0.29A10+ 0.19A11+ 0.16A12+ 0.42A13+ 2.95A14+ 14A15+ 0.12A16+ 0.15A17+ 0.18A18+ 0.11A19+ 0.24A20+ 0.27A21 \leq 50 \dots \dots \dots (47)$

Constraint of the Lower limit of the calcium percentage in the initial mash:

• $0.05A1+ 0.08A2+ 0.07A3+ 0.04A4+ 0.09A5+ 0.16A6+ 0.15A7+ 0.45A8+ 0.32A10+ 0.26A11+ 0.38A12+ 2.02A13+ 5.02A14+ 30A15+ 0.17A16+ 0.13A17+ 0.2A18+ 0.52A19+ 1.3A20+ 1.5A21+ 38A22 \geq 95 \dots \dots \dots (48)$

Constraint of the upper limit of the calcium percentage in the initial mash:

• $0.05A1+ 0.08A2+ 0.07A3+ 0.04A4+ 0.09A5+ 0.16A6+ 0.15A7+ 0.45A8+ 0.32A10+ 0.26A11+ 0.38A12+ 2.02A13+ 5.02A14+ 30A15+ 0.17A16+ 0.13A17+ 0.2A18+ 0.52A19+ 1.3A20+ 1.5A21+ 38A22 \leq 110 \dots \dots \dots (49)$

Constraint of the lower limit of the sodium percentage in the initial mash

• $0.07A1+ 0.02A2+ 0.01A3+ 0.01A4+ 0.11A5+ 0.3A6+ 0.07A7+ 0.1A8+ 0.03A9+ 0.24A10+ 0.34A11+ 0.3A13+ 0.46A15+ 0.0816+ 0.18A20+ 0.19A21+ 50A27 \geq 20 \dots \dots \dots (50)$

Constraint of the upper limit of the sodium percentage in the initial mash

$$\bullet 0.07A1+ 0.02A2+ 0.01A3+ 0.01A4+ 0.11A5+ 0.3A6+ 0.07A7+ 0.1A8+ 0.03A9+ 0.24A10+ 0.34A11+ 0.3A13+ 0.46A15+ 0.0816+ 0.18A20+ 0.19A21+ 50A27 \leq 25 \dots \dots \dots (51)$$

Constraint of the lower limit of lysine amino acid percentage in the initial mash:

$$\bullet 0.39A1+ 0.39A2+ 0.24A3+ 0.25A4+ 0.3A5+ 0.53A6+ 0.59A7+ 0.73A8+ 1.29A9+ 2.91A10+ 3.17A11+ 1.73A12+ 1.09A13+ 4.83A14+ 0.87A15+ 1.55A16+ 1.3A17+ 1.34A18+ 1.73A19+ 0.82A20+ 0.9A21 \geq 100 \dots \dots \dots (52)$$

Constraint of the upper limit of lysine amino acid percentage in the initial mash:

$$\bullet 0.39A1+ 0.39A2+ 0.24A3+ 0.25A4+ 0.3A5+ 0.53A6+ 0.59A7+ 0.73A8+ 1.29A9+ 2.91A10+ 3.17A11+ 1.73A12+ 1.09A13+ 4.83A14+ 0.87A15+ 1.55A16+ 1.3A17+ 1.34A18+ 1.73A19+ 0.82A20+ 0.9A21 \leq 123 \dots \dots \dots (53)$$

Constraint of the lower limit of methionine amino acid percentage in the initial mash:

$$\bullet 0.37A1+ 0.37A2+ 0.4A3+ 0.35A4+ 0.3A5+ 0.42A6+ 0.47A7+ 1.91A8+ 2.79A9+ 1.33A10+ 1.47A11+ 2.22A12+ 1.86A13+ 2.32A14+ 0.29A15+ 0.47A16+ 0.48A17+ 0.59A18+ 0.41A19+ 0.51A20+ 0.55A21 \geq 70 \dots \dots \dots (54)$$

Constraint of the lower limit of methionine amino acid percentage in the initial mash:

$$\bullet 0.37A1+ 0.37A2+ 0.4A3+ 0.35A4+ 0.3A5+ 0.42A6+ 0.47A7+ 1.91A8+ 2.79A9+ 1.33A10+ 1.47A11+ 2.22A12+ 1.86A13+ 2.32A14+ 0.29A15+ 0.47A16+ 0.48A17+ 0.59A18+ 0.41A19+ 0.51A20+ 0.55A21 \leq 90 \dots \dots \dots (55)$$

Constraint of the lower limit of the thiamin amount in the initial mash

$$\bullet 4.4A1+ 5.1A2+ 4A3+ 3.9A4+ 0.6A5+ 7.9A6+ 2.6A7+ 0.2A8+ 0.2A9+ 4A10+ 6.6A11+ 2.86A13+ 0.2A14+ 3.6A15+ 4A20+ 0.9A21 \geq 180 \dots \dots \dots (56)$$

Constraint of the upper limit of the thiamin amount in the initial mash

$$\bullet 4.4A1+ 5.1A2+ 4A3+ 3.9A4+ 0.6A5+ 7.9A6+ 2.6A7+ 0.2A8+ 0.2A9+ 4A10+ 6.6A11+ 2.86A13+ 0.2A14+ 3.6A15+ 4A20+ 0.9A21 \leq 220 \dots \dots \dots (57)$$

Constraint of the lower limit of the ribophlavine amount in the initial mash:

$$\bullet 1.2A1+ 2A2+ 1.1A3+ 1.2A4+ 0.6A5+ 3.1A6+ 0.9A7+ 1.5A8+ 2.2A9+ 2.6A10+ 3.3A11+ 3.3A12+ 3.74A13+ 4.6A14+ 18.9A15+ 44.4A19+ 15.4A20+ 2.4A21 \geq 220 \dots \dots \dots (58)$$

Constraint of the upper limit of the ribophlavine amount in the initial mash:

- $1.2A1+ 2A2+ 1.1A3+ 1.2A4+ 0.6A5+ 3.1A6+0.9A7+1.5A8+2.2A9+2.6A10 +3.3A11+3.3A12 +3.7A13+ 4.6A14+18.9A15+44.4A19+15.4A20+2.4A21 \leq 360 \dots \dots \dots (59)$

Constraint of the lower limit of the pantothenic amount in the initial mash:

- $12.1A1+ 6.5A2+ 5A3+ 11.4A4+ 3.3A5+ 29A6+ 13A7+ 10.3A8+ 2.9A9+ 15.2A10+ 14.5A11+ 10.1A12+ 6.38A13+ 3.3A14+ 33.9A15+ 82.9A19+ 32.8A20+ 39A21 \geq 950 \dots \dots \dots (60)$

Constraint of the upper limit of the pantothenic amount in the initial mash:

- $12.1A1+ 6.5A2+ 5A3+ 11.4A4+ 3.3A5+ 29A6+ 13A7+ 10.3A8+ 2.9A9+ 15.2A10+ 14.5A11+ 10.1A12+ 6.38A13+ 3.3A14+ 33.9A15+ 82.9A19+ 32.8A20+ 39A21 \leq 1000 \dots \dots \dots (61)$

Constraint of the lower limit of the niacin amount in the initial mash:

- $56.6A1+ 57.4A2+ 22.9A3+ 42.7A4+ 14.1A5+ 209A6+ 20A7+ 49.9A8+ 5.5A9+ 30.4A10+ 26.8A11+ 220A12+ 30.8A13+ 47.1A14+ 10.6A15+ 500A19+ 54.6A20+ 46A21 \geq 3400 \dots \dots (62)$

Constraint of the upper limit of the niacin amount in the initial mash:

- $56.6A1+ 57.4A2+ 22.9A3+ 42.7A4+ 14.1A5+ 209A6+ 20A7+ 49.9A8+ 5.5A9+ 30.4A10+ 26.8A11+ 220A12+ 30.8A13+ 47.1A14+ 10.6A15+ 500A19+ 54.6A20+ 46A21 \leq 3600 \dots \dots (63)$

Constraint of the upper limit for the ratio of (energy/protein) in the initial mash:

- $250A1+ 305.56A2+ 386.47A3+ 281.58A4+ 385A5+ 88.65A6+ 106.45A7+ 79.4A8+ 62.66A9+ 51.11A10+ 50A11+ 41.79A12+ 54.29A13+ 45.8A14+ 115.53A15+ 96.55A16+ 97.08A17+ 132.5A18+ 112.64A19+ 83.98A20+ 74.88A21 \leq 220 \dots \dots \dots (64)$

Constraint of vegetable protein sources in the initial mash:

- $A8+ A9+ A10+ A12+ A13+ A16+ A17+ A18+ A19+ A20+ A21 \leq 35 \dots \dots \dots (65)$

3.5. The mathematical model for the grower mash

The objective function is to minimize the summation of the cost of 100 kgm of the grower mash

- $Min z = 510B1+ 430B2+ 490B3+ 410B4+ 290B5+ 400B6+ 400B7+ 958B8+ 1450B9+ 838B10+ 910B11+ 460B12+ 3100B13+ 1750B14+ 850B15+ 510B16+ 1210B17+ 710B18+ 710B19+ 610B20+ 610B21+ 110B22+ 110B23+ 1810B24+ 1930B25+ 1750B26+ 110B27 \dots \dots \dots (66)$

The objective function subject to the following constraints:

- *Constraint of the wheat in the grower mash:* $B1 \leq 70 \dots \dots \dots (67)$

- *Constraint of the barely in the grower mash:* $B2 \leq 20 \dots \dots \dots (68)$

- *Constraint of the maiz in the grower mash: $B3 \leq 70$...* (69)
- *Constraint of the white corn in the grower mash: $B4 \leq 10$...* (70)
- *Constraint of the rice in the grower mash: $B5 \leq 30$...* (71)
- *Constraint of the coarse wheat rane in the grower mash: $B6 \leq 5$...* (72)
- *Constraint of the fine wheat brane in the grower mash: $B7 \leq 5$...* (73)
- *Constraint of the maiz gluten 42% protein in the grower mash: $B8 \leq 50$...* (74)
- *Constraint of the maiz gluten 60% protein in the grower mash: $B9 \leq 5$...* (75)
- *Constraint of the soybean meal 44% protein in the grower mash: $B10 \leq 300$...* (76)
- *Constraint of the soybean meal 49% protein in the grower mash: $B11 \leq 20$...* (77)
- *Constraint of the sunflower meal in the grower mash: $B12 \leq 40$...* (78)
- *constraint of the sesame meal in the grower mash: $B13 \leq 30$...* (79)
- *Constraint of the fish meal in the grower mash: $B14 \leq 5$...* (80)
- *constraint of the bone meal in the grower mash: $B15 \leq 2.5$...* (81)
- *Constraint of the bean in the grower mash: $B16 \leq 10$...* (82)
- *Constraint of the white beans in the grower mash: $B17 < 10$...* (83)
- *of the chickpeas in the grower mash: $B18 \leq 10$...* (84)
- *constraint of the lentis in the grower mash: $B19 \leq 10$...* (85)
- *Constraint of the dehydrated alfalfa 17% protein in the grower mash: $B20 \leq 7$...* (86)
- *Constraint of the dehydrated alfalfa leavs 20% protein in the growe mash: $B21 \leq 7$...* (87)
- *Constraint of the limestone in the grower mash: $B22 \leq 1$...* (88)
- *Constraints of the vegetable oils in the grower mash: $B23 \leq 4$...* (89)
- *Constraints of the crude vegetable oils in the grower mash: $B24 \leq 4$...* (90)
- *Constraints of the oils in the grower mash: $B23+B24 \leq 4$...* (91)
- *Constraint of the vegetable protein concentrates & vitamins in the grower mash: $B25 \leq 10$...* (92)
- *Constraint of the animal protein concentrates & vitamins in the grower mash: $B26 \leq 10$...* (93)
- *Constraint of sodium chloride in the grower mash: $B27 \leq 0.25$...* (94)

Constraint of all ingredients in grower mash:

- *$B1+ B2+ B3+ B4+ B5+ B6+ B7+ B8+ B9+ B10+ B11+ B12+ B13+ B15+ B16+ B17+ B18+ B19+ B20+ B21+ B22+ B23+ B24+ B25+ B26+ B27= 100$...* (95)
- *Constraint of the upper limit of the legumes in the grower mash: $B16+B17+B18+B19 >= 3$.* (96)
- *Constraint of the lower limit of the legumes in the grower mash: $B16+B17+B18+B19 <= 10$.* (97)
- *Constraint of the meals in the grower mash: $B8+B9+B10+B11+B12+B13 <= 40$...* (98)

Constraint of the grain and another energy sources in the grower mash:

- *$B1+ B2+ B3+ B4+ B5+ B6+ B7 <= 500$...* (99)
- *Constraint of the oils in the grower mash: $B23+B24 <= 4$...* (100)

Constraint of the upper limit of the concentrated vegetable protein in the grower mash:

- *$B25+B26 <= 10$...* (101)

Constraint of protein percentage in the grower mash:

$$\bullet 13B1+ 9B2+ 8.72B3+ 11.4B4+ 8.18B5+ 14.1B6+ 15.5B7+ 42B8+ 60B9+ 45B10+ 50B11+ 46.9B12+ 44.5B13+ 64.7B14+ 12.1B15+ 26.1B16+ 24B17+ 20.8B18+ 23.5B19+ 17.3B20+ 21.1B21 = 2200\dots\dots\dots(102)$$

Constraint of the metabolizable energy in the grower mash:

$$\bullet 3250B1+ 2750B2+ 3370B3+ 3210B4+ 3150B5+ 1250B6+ 1650B7+ 3335B8+ 3760B9+ 2300B10 + 2500B11+ 1960B12+ 2416B13+ 2963B14+ 1398B15+ 2520B16+ 2330B17+ 2756B18+ 2647B19+ 1453B20+ 1580B21+ 7010B23+ 8900B24= 300000\dots\dots\dots(103)$$

Constraint of the upper limit of the lipids percentage in the grower mash:

$$\bullet 1.9B1+ 1.9B2+ 3.9B3+ 2.8B4+ 1.9B5+ 4.1B6+ 2.94B7+ 2.3B8+ 1.7B9+ 4.7B10+ 0.8B11+ 5.1B12+ 2.8B13+ 4.4B14+ 3.2B15+ 1.5B16+ 2.6B20+ 3.6B21+ 90B23+ 90B24 \leq 700\dots\dots(104)$$

Constraint of the lower limit of the lipid percentage in the grower mash:

$$\bullet 1.9B1+ 1.9B2+ 3.9B3+ 2.8B4+ 1.9B5+ 4.1B6+ 2.94B7+ 2.3B8+ 1.7B9+ 4.7B10+ 0.8B11+ 5.1B12+ 2.8B13+ 4.4B14+ 3.2B15+ 1.5B16+ 2.6B20+ 3.6B21+ 90B23+ 90B24 \geq 390\dots\dots(105)$$

Constraint of the upper limit of the humidity percentage in the grower mash:

$$\bullet 11B1+ 11B2+ 12B3+ 11B4+ 11B5+ 11B6+ 11B7+ 9B8+ 9B9+ 10.4B10+ 10B11+ 7B12+ 10B13+ 8B14+ 5B15+ 13.4B16+ 7B20+ 10.3B21 \leq 1000\dots\dots\dots(106)$$

Constraint of the lower limit of the humidity percentage in the grower mash:

$$\bullet 11B1+ 11B2+ 12B3+ 11B4+ 11B5+ 11B6+ 11B7+ 9B8+ 9B9+ 10.4B10+ 10B11+ 7B12+ 10B13+ 8B14+ 5B15+ 13.4B16+ 7B20+ 10.3B21 > 500\dots\dots\dots(107)$$

Constraint of upper limit of the crude fiber percentage in the grower mash:

$$\bullet 2.4B1+ 5B2+ 2B3+ 2B4+ 9B5+ 10B6+ 3B7+ 4B8+ 1.3B9+ 6B10+ 3B11+ 5B12+ 14B13+ B14+ 2B15+ 5.7B16+ 24.8B20+ 20.1B21 \leq 700\dots\dots\dots(108)$$

Constraint of lower limit of the crude fiber percentage in the grower mash:

$$\bullet 2.4B1+ 5B2+ 2B3+ 2B4+ 9B5+ 10B6+ 3B7+ 4B8+ 1.3B9+ 6B10+ 3B11+ 5B12+ 14B13+ B14+ 2B15+ 5.7B16+ 24.8B20+ 20.1B21 > 300\dots\dots\dots(109)$$

Constraint of the lower limit of the ash in the grower mash:

$$\bullet 1.6B1+ 2.4B2+ 1.8B3+ 1.7B4+ 4.5B5+ 6.1B6+ 2.1B7+ 2.4B8+ 2B9+ 5.7B10+ 5.6B11+ 9.3B12+ 7.1B13+ 21.7B14+ 71.8B15+ 3.6B16+ 9B20+ 10.3B21 \geq 200 \dots \dots \dots (110)$$

Constraint of the upper limit of the ash in the grower mash:

$$\bullet 1.6B1+ 2.4B2+ 1.8B3+ 1.7B4+ 4.5B5+ 6.1B6+ 2.1B7+ 2.4B8+ 2B9+ 5.7B10+ 5.6B11+ 9.3B12+ 7.1B13+ 21.7B14+ 71.8B15+ 3.6B16+ 9B20+ 10.3B21 \leq 500 \dots \dots \dots (111)$$

Constraint of the lower limit of the phosphor percentage in the grower mash:

$$\bullet 0.12B1+ 0.16B2+ 0.11B3+ 0.13B4+ 0.04B5+ 0.34B6+ 0.37B7+ 0.15B8+ 0.19B9+ 0.29B10+ 0.19B11+ 0.16B12+ 0.42B13+ 2.95B14+ 14B15+ 0.12B16+ 0.15B17+ 0.18B18+ 0.11B19+ 0.24B20+ 0.27B21 \geq 35 \dots \dots \dots (112)$$

Constraint of the upper limit of the phosphor percentage in the grower mash:

$$\bullet 0.12B1+ 0.16B2+ 0.11B3+ 0.13B4+ 0.04B5+ 0.34B6+ 0.37B7+ 0.15B8+ 0.19B9+ 0.29B10+ 0.19B11+ 0.16B12+ 0.42B13+ 2.95B14+ 14B15+ 0.12B16+ 0.15B17+ 0.18B18+ 0.11B19+ 0.24B20+ 0.27B21 \leq 40 \dots \dots \dots (113)$$

Constraint of the lower limit of the calcium percentage in the grower mash:

$$\bullet 0.05B1+ 0.08B2+ 0.07B3+ 0.04B4+ 0.09B5+ 0.16B6+ 0.15B7+ 0.45B8+ 0.32B10+ 0.26B11+ 0.38B12+ 2.02B13+ 5.02B14+ 30B15+ 0.17B16+ 0.13B17+ 0.2B18+ 0.52B19+ 1.3B20+ 1.5B21+ 38B22 \geq 87 \dots \dots \dots (114)$$

Constraint of the upper limit of the calcium percentage in the grower mash:

$$\bullet 0.05B1+ 0.08B2+ 0.07B3+ 0.04B4+ 0.09B5+ 0.16B6+ 0.15B7+ 0.45B8+ 0.32B10+ 0.26B11+ 0.38B12+ 2.02B13+ 5.02B14+ 30B15+ 0.17B16+ 0.13B17+ 0.2B18+ 0.52B19+ 1.3B20+ 1.5B21+ 38B22 \leq 92 \dots \dots \dots (115)$$

Constraint of the upper limit of the sodium percentage in the grower mash:

$$\bullet 0.07B1+ 0.02B2+ 0.01B3+ 0.01B4+ 0.11B5+ 0.3B6+ 0.07B7+ 0.1B8+ 0.03B9+ 0.24B10+ 0.34B11+ 0.3B13+ 0.46B15+ 0.0816+ 0.18B20+ 0.19B21 \leq 22 \dots \dots \dots (116)$$

Constraint of the lower limit of the sodium percentage in the grower mash:

$$\bullet 0.07B1+ 0.02B2+ 0.01B3+ 0.01B4+ 0.11B5+ 0.3B6+ 0.07B7+ 0.1B8+ 0.03B9+ 0.24B10+ 0.34B11+ 0.3B13+ 0.46B15+ 0.0816+ 0.18B20+ 0.19B21+ 50B27 \geq 14 \dots \dots \dots (117)$$

Constraint of the lower limit of the lysine amino acid percentage in the grower mash:

$$\bullet 0.39B1+ 0.39B2+ 0.24B3+ 0.25B4+ 0.3B5+ 0.53B6+ 0.59B7+ 0.73B8+ 1.29B9+ 2.91B10+ 3.17B11+ 1.73B12+ 1.09B13+ 4.83B14+ 0.87B15+ 1.55B16+ 1.3B17+ 1.34B18+ 1.73B19+ 0.82B20+ 0.9B21 \geq 76 \dots \dots \dots (118)$$

Constraint of the upper limit of the lysine amino acid percentage in the grower mash:

$$\bullet 0.39B1+ 0.39B2+ 0.24B3+ 0.25B4+ 0.3B5+ 0.53B6+ 0.59B7+ 0.73B8+ 1.29B9+ 2.91B10+ 3.17B11+ 1.73B12+ 1.09B13+ 4.83B14+ 0.87B15+ 1.55B16+ 1.3B17+ 1.34B18+ 1.73B19+ 0.82B20+ 0.9B21 \leq 114 \dots \dots \dots (119)$$

Constraint of the upper limit of the methyonine amino acid percentage in the grower mash:

$$\bullet 0.37B1+ 0.37B2+ 0.4B3+ 0.35B4+ 0.3B5+ 0.42B6+ 0.47B7+ 1.91B8+ 2.79B9+ 1.33B10+ 1.47B11+ 2.22B12+ 1.86B13+ 2.32B14+ 0.29B15+ 0.47B16+ 0.48B17+ 0.59B18+ 0.41B19+ 0.51B20+ 0.55B21 \leq 76 \dots \dots \dots (120)$$

Constraint of the lower limit of the methyonine amino acid percentage in the grower mash:

$$\bullet 0.37B1+ 0.37B2+ 0.4B3+ 0.35B4+ 0.3B5+ 0.42B6+ 0.47B7+ 1.91B8+ 2.79B9+ 1.33B10+ 1.47B11+ 2.22B12+ 1.86B13+ 2.32B14+ 0.29B15+ 0.47B16+ 0.48B17+ 0.59B18+ 0.41B19+ 0.51B20+ 0.55B21 \geq 32 \dots \dots \dots (121)$$

Constraint of the lower limit of the thiamin amount in the grower mash:

$$\bullet 4.4B1+ 5.1B2+ 4B3+ 3.9B4+ 0.6B5+ 7.9B6+ 2.6B7+ 0.2B8+ 0.2B9+ 4B10+ 6.6B11+ 2.86B13+ 0.2B14+ 3.6B15+ 4B20+ 0.9B21 \geq 160 \dots \dots \dots (122)$$

Constraint of the lower limit of the thiamin amount in the grower mash:

$$\bullet 4.4B1+ 5.1B2+ 4B3+ 3.9B4+ 0.6B5+ 7.9B6+ 2.6B7+ 0.2B8+ 0.2B9+ 4B10+ 6.6B11+ 2.86B13+ 0.2B14+ 3.6B15+ 4B20+ 0.9B21 \leq 322 \dots \dots \dots (123)$$

Constraint of the lower limit of the riboflavin amount in the grower mash

$$\bullet 1.2B1+ 2B2+ 1.1B3+ 1.2B4+ 0.6B5+ 3.1B6+ 0.9B7+ 1.5B8+ 2.2B9+ 2.6B10+ 3.3B11+ 3.3B12+ 3.74B13+ 4.6B14+ 18.9B15+ 44.4B19+ 15.4B20+ 2.4B21 \geq 220 \dots \dots \dots (124)$$

Constraint of the upper limit of the riboflavin amount in the grower mash:

$$\bullet 1.2B1+ 2B2+ 1.1B3+ 1.2B4+ 0.6B5+ 3.1B6+ 0.9B7+ 1.5B8+ 2.2B9+ 2.6B10+ 3.3B11+ 3.3B12+ 3.74B13+ 4.6B14+ 18.9B15+ 44.4B19+ 15.4B20+ 2.4B21 \leq 350 \dots \dots \dots (125)$$

Constraint of the lower limit of the pantothenic amount in the grower mash:

$$\bullet 12.1B1+ 6.5B2+ 5B3+ 11.4B4+ 3.3B5+ 29B6+ 13B7+ 10.3B8+ 2.9B9+ 15.2B10+ 14.5B11+ 10.1B12+ 6.38B13+ 3.3B14+ 33.9B15+ 82.9B19+ 32.8B20+ 39B21 \geq 1000 \dots \dots \dots (126)$$

Constraint of the upper limit of the pantothenic amount in the grower mash:

$$\bullet 12.1B1+ 6.5B2+ 5B3+ 11.4B4+ 3.3B5+ 29B6+ 13B7+ 10.3B8+ 2.9B9+ 15.2B10+ 14.5B11+ 10.1B12+ 6.38B13+ 3.3B14+ 33.9B15+ 82.9B19+ 32.8B20+ 39B21 \leq 1100 \dots \dots \dots (127)$$

Constraint of the lower limit of the niacin amount in the grower mash:

$$\bullet 56.6B1+ 57.4B2+ 22.9B3+ 42.7B4+ 14.1B5+ 209B6+ 20B7+ 49.9B8+ 5.5B9+ 30.4B10+ 26.8B11+ 220B12+ 30.8B13+ 47.1B14+ 10.6B15+ 500B19+ 54.6B20+ 46B21 \geq 3500 \dots \dots (128)$$

Constraint of the upper limit of the niacin amount in the grower mash:

$$\bullet 56.6B1+ 57.4B2+ 22.9B3+ 42.7B4+ 14.1B5+ 209B6+ 20B7+ 49.9B8+ 5.5B9+ 30.4B10+ 26.8B11+ 220B12+ 30.8B13+ 47.1B14+ 10.6B15+ 500B19+ 54.6B20+ 46B21 \leq 3750 \dots (129)$$

Constraint of vegetable protein sources in the grower mash:

$$\bullet B8+ B9+ B10+ B12+ B13+ B16+ B17+ B18+ B19+ B20+ B21 \leq 35 \dots \dots \dots (130)$$

3.6. *The mathematical model for the finisher mash*

The objective function is to minimize the summation of the cost of 100 kgm of the finisher mash

$$\bullet \text{Min } z = 510C1+ 430C2+ 490C3+ 410C4+ 290C5+ 400C6+ 4000C7+ 958C8+ 1450C9+ 838C10+ 910C11+ 460C12+ 3100C13+ 1750C14+ 850C15+ 510C16+ 1210C17+ 710C18+ 710C19+ 610C20+ 610C21+ 110C22+ 110C23+ 1810C24+ 1930C25+ 1750C26+ 110C27 \dots \dots \dots (131)$$

The objective function subject to the following constraints

- *constraint of the wheat in the finisher mash: $C1 \leq 70 \dots \dots \dots (132)$*
- *constraint of the barely in the finisher mash: $C2 \leq 20 \dots \dots \dots (133)$*
- *constraint of the maiz in the finisher mash: $C3 \leq 70 \dots \dots \dots (134)$*
- *constraint of the white corn in the finisher mash: $C4 \leq 10 \dots \dots \dots (135)$*
- *Constraint of the rice in the finisher mash: $C5 \leq 30 \dots \dots \dots (136)$*
- *Constraint of the coarse wheat bran in the finisher mash: $C6 \leq 5 \dots \dots \dots (137)$*
- *Constraint of the fine wheat bran in the finisher mash: $C7 \leq 5 \dots \dots \dots (138)$*
- *Constraint of the maiz gluten 42% protein in the finisher mash: $C8 \leq 50 \dots \dots \dots (139)$*
- *Constraint of the maiz gluten 60% protein in the finisher mash: $C9 \leq 5 \dots \dots \dots (140)$*
- *Constraint of the soybean meal 44% protein in the finisher mash: $C10 \leq 30 \dots \dots \dots (141)$*
- *Constraint of the soybean meal 49% protein in the finisher mash: $C11 \leq 20 \dots \dots \dots (142)$*
- *Constraint of the sunflower meal in the finisher mash: $C12 \leq 40 \dots \dots \dots (143)$*
- *constraint of the sesame meal in the finisher mash: $C13 \leq 30 \dots \dots \dots (144)$*

- *Constraint of the fish meal in the finisher mash: $C14 \leq 5$(145)*
- *constraint of the bone meal in the finisher mash: $C15 \leq 2.5$(146)*
- *Constraint of the bean in the finisher mash: $C16 < 10$(147)*
- *Constraint of the white beans in the finisher mash: $C17 \leq 10$(148)*
- *of the chickpeas in the finisher mash: $C18 \leq 10$(149)*
- *constraint of the lentis in the finisher mash: $C19 \leq 10$ (150)*
- *Constraint of dehydrated alfalfa 17% protein in the finisher mash: $C20 \leq 7$(151)*
- *Constraint of the dehydrated alfalfa leaves 20% protein in the grower mash: $C21 \leq 7$(152)*
- *Constraint of the limestone in the finisher mash: $C22 \leq 1$(153)*
- *Constraints of the vegetable oils in the finisher mash: $C23 \leq 4$(154)*
- *Constraints of the crude vegetable oils in the finisher mash: $C24 \leq 4$(155)*
- *Constraint of the vegetable protein concentrates & vitamins in the finisher mash: $C25 < 10$(156)*
- *Constraint of the animal protein concentrates & vitamins in the finisher mash: $C26 \leq 10$(157)*
- *Constraint of the sodium chloride in the finisher mash: $C27 \leq 0.25$(158)*

Constraint of all ingredients in finisher mash:

- *$C1 + C2 + C3 + C4 + C5 + C6 + C7 + C8 + C9 + C10 + C11 + C12 + C13 + C15 + C16 + C17 + C18 + C19 + C20 + C21 + C22 + C23 + C24 + C25 + C26 + C27 = 100$(159)*
- *Constraint of the upper limit of the legumes in the finisher mash: $C16 + C17 + C18 + C19 \geq 3$(160)*

Constraint of the lower limit of the legumes in the finisher mash:

- *$C16 + C17 + C18 + C19 \leq 10$(159)*
- *Constraint of the meals in the finisher mash: $C8 + C9 + C10 + C11 + C12 + C13 \leq 40$(161)*

Constraint of the grain and another energy sources in the finisher mash:

- *$C1 + C2 + C3 + C4 + C5 + C6 + C7 \leq 500$(162)*
- *Constraint of the oils in the finisher mash: $C23 + C24 \leq 4$(163)*

Constraint of the upper limit of the concentrated vegetable protein in the finisher mash:

- *$C25 + C26 \leq 10$(164)*

Constraint of protein percentage in the finisher mash

- *$13C1 + 9C2 + 8.72C3 + 11.4C4 + 8.18C5 + 14.1C6 + 15.5C7 + 42C8 + 60C9 + 45C10 + 50C11 + 46.9C12 + 44.5C13 + 64.7C14 + 12.1C15 + 26.1C16 + 24C17 + 20.8C18 + 23.5C19 + 17.3C20 + 21.1C21 = 2000$(165)*

Constraint of the metabolizable energy in the finisher mash:

$$\bullet 3250C1+ 2750C2+ 3370C3+ 3210C4+ 3150C5+ 1250C6+ 1650C7+ 3335C8+ 3760C9+ 2300C10+ 2500C11+ 1960C12+ 2416C13+ 2963C14+ 1398C15+ 2520C16+ 2330C17+ 2756C18+ 2647C19+ 1453C20+ 1580C21+ 7010C23+ 8900C24= 320000\dots\dots\dots(166)$$

Constraint of the upper limit of the lipid percentage in the finisher mash:

$$\bullet 1.9C1+ 1.9C2+ 3.9C3+ 2.8C4+ 1.9C5+ 4.1C6+ 2.94C7+ 2.3C8+ 1.7C9+ 4.7C10+ 0.8C11+ 5.1C12+ 2.8C13+ 4.4C14+ 3.2C15+ 1.5C16+ 2.6C20+ 3.6C21+ 90C23+ 90C24\leq 700\dots(167)$$

Constraint of the lower limit of the lipid percentage in the finisher mash:

$$\bullet 1.9C1+ 1.9C2+ 3.9C3+ 2.8C4+ 1.9C5+ 4.1C6+ 2.94C7+ 2.3C8+ 1.7C9+ 4.7C10+ 0.8C11+ 5.1C12+ 2.8C13+ 4.4C14+ 3.2C15+ 1.5C16+ 2.6C20+ 3.6C21+ 90C23+ 90C24\geq 310\dots\dots(168)$$

Constraint of the upper limit of the humidity percentage in the finisher mash:

$$\bullet 11C1+ 11C2+ 12C3+ 11C4+ 11C5+ 11C6+ 11C7+ 9C8+ 9C9+ 10.4C10+ 10C11+ 7C12+ 10C13+ 8C14+ 5C15+ 13.4C16+ 7C20+ 10.3C21\leq 1000\dots\dots\dots(169)$$

Constraint of the lower limit of the humidity percentage in the finisher mash

$$\bullet 11C1+ 11C2+ 12C3+ 11C4+ 11C5+ 11C6+ 11C7+ 9C8+ 9C9+ 10.4C10+ 10C11+ 7C12+ 10C13+ 8C14+ 5C15+ 13.4C16+ 7C20+ 10.3C21\geq 500\dots\dots\dots(170)$$

Constraint of upper limit of the crude fiber percentage in the finisher mash

$$\bullet 2.4C1+ 5C2+ 2C3+ 2C4+ 9C5+ 10C6+ 3C7+ 4C8+ 1.3C9+ 6C10+ 3C11+ 5C12+ 14C13+ C14+ 2C15+ 5.7C16+ 24.8C20+ 20.1C21\leq 600\dots\dots\dots(171)$$

Constraint of lower limit of the crude fiber percentage in the finisher mash

$$\bullet 2.4C1+ 5C2+ 2C3+ 2C4+ 9C5+ 10C6+ 3C7+ 4C8+ 1.3C9+ 6C10+ 3C11+ 5C12+ 14C13+ C14+ 2C15+ 5.7C16+ 24.8C20+ 20.1C21\geq 300\dots\dots\dots(172)$$

Constraint of the lower limit of the ash in the finisher mash

$$\bullet 1.6C1+ 2.4C2+ 1.8C3+ 1.7C4+ 4.5C5+ 6.1C6+ 2.1C7+ 2.4C8+ 2C9+ 5.7C10+ 5.6C11+ 9.3C12+ 7.1C13+ 21.7C14+ 71.8C15+ 3.6C16+ 9C20+ 10.3C21\geq 100\dots\dots\dots(173)$$

Constraint of the upper limit of the ash in the finisher mash

$$\bullet 1.6C1+ 2.4C2+ 1.8C3+ 1.7C4+ 4.5C5+ 6.1C6+ 2.1C7+ 2.4C8+ 2C9+ 5.7C10+ 5.6C11+ 9.3C12+ 7.1C13+ 21.7C14+ 71.8C15+ 3.6C16+ 9C20+ 10.3C21\leq 500\dots\dots\dots(174)$$

Constraint of the upper limit of the phosphor percentage in the finisher mash

$$\bullet 0.12C1+ 0.16C2+ 0.11C3+ 0.13C4+ 0.04C5+ 0.34C6+ 0.37C7+ 0.15C8+ 0.19C9+ 0.29C10+ 0.19C11+ 0.16C12+ 0.42C13+ 2.95C14+ 14C15+ 0.12C16+ 0.15C17+ 0.18C18+ 0.11C19+ 0.24C20+ 0.27C21 \leq 35 \dots \dots \dots (175)$$

Constraint of the lower limit of the phosphor percentage in the finisher mash

$$\bullet 0.12C1+ 0.16C2+ 0.11C3+ 0.13C4+ 0.04C5+ 0.34C6+ 0.37C7+ 0.15C8+ 0.19C9+ 0.29C10+ 0.19C11+ 0.16C12+ 0.42C13+ 2.95C14+ 14C15+ 0.12C16+ 0.15C17+ 0.18C18+ 0.11C19+ 0.24C20+ 0.27C21 \geq 19 \dots \dots \dots (176)$$

Constraint of the lower limit of the calcium percentage in the finisher mash

$$\bullet 0.05C1+ 0.08C2+ 0.07C3+ 0.04C4+ 0.09C5+ 0.16C6+ 0.15C7+ 0.45C8+ 0.32C10+ 0.26C11+ 0.38C12+ 2.02C13+ 5.02C14+ 30C15+ 0.17C16+ 0.13C17+ 0.2C18+ 0.52C19+ 1.3C20+ 1.5C21+ 38C22 \geq 33 \dots \dots \dots (177)$$

Constraint of the upper limit of the calcium percentage in the finisher mash

$$\bullet 0.05C1+ 0.08C2+ 0.07C3+ 0.04C4+ 0.09C5+ 0.16C6+ 0.15C7+ 0.45C8+ 0.32C10+ 0.26C11+ 0.38C12+ 2.02C13+ 5.02C14+ 30C15+ 0.17C16+ 0.13C17+ 0.2C18+ 0.52C19+ 1.3C20+ 1.5C21+ 38C22 \leq 90 \dots \dots \dots (178)$$

Constraint of the upper limit of the sodium percentage in the finisher mash

$$\bullet 0.07C1+ 0.02C2+ 0.01C3+ 0.01C4+ 0.11C5+ 0.3C6+ 0.07C7+ 0.1C8+ 0.03C9+ 0.24C10+ 0.34C11+ 0.3C13+ 0.46C15+ 0.0816+ 0.18C20+ 0.19C21+ 50C27 \leq 19 \dots \dots \dots (179)$$

Constraint of the lower limit of the sodium percentage in the finisher mash

$$\bullet 0.07C1+ 0.02C2+ 0.01C3+ 0.01C4+ 0.11C5+ 0.3C6+ 0.07C7+ 0.1C8+ 0.03C9+ 0.24C10+ 0.34C11+ 0.3C13+ 0.46C15+ 0.0816+ 0.18C20+ 0.19C21+ 50C27 \geq 12 \dots \dots \dots (180)$$

Constraint of the lower limit of the lysine amino acid percentage in the finisher mash

$$\bullet 0.39C1+ 0.39C2+ 0.24C3+ 0.25C4+ 0.3C5+ 0.53C6+ 0.59C7+ 0.73C8+ 1.29C9+ 2.91C10+ 3.17C11+ 1.73C12+ 1.09C13+ 4.83C14+ 0.87C15+ 1.55C16+ 1.3C17+ 1.34C18+ 1.73C19+ 0.82C20+ 0.9C21 \geq 85 \dots \dots \dots (181)$$

Constraint of the upper limit of the lysine amino acid percentage in the finisher mash

$$\bullet 0.39C1+ 0.39C2+ 0.24C3+ 0.25C4+ 0.3C5+ 0.53C6+ 0.59C7+ 0.73C8+ 1.29C9+ 2.91C10+ 3.17C11+ 1.73C12+ 1.09C13+ 4.83C14+ 0.87C15+ 1.55C16+ 1.3C17+ 1.34C18+ 1.73C19+ 0.82C20+ 0.9C21 \leq 95 \dots \dots \dots (182)$$

Constraint of the lower limit of the methoyonine amino acid percentage in the finisher mash

$$\bullet 0.37C1+ 0.37C2+ 0.4C3+ 0.35C4+ 0.3C5+ 0.42C6+ 0.47C7+ 1.91C8+ 2.79C9+ 1.33C10+ 1.47C11+ 2.22C12+ 1.86C13+ 2.32C14+ 0.29C15+ 0.47C16+ 0.48C17+ 0.59C18+ 0.41C19+ 0.51C20+ 0.55C21 \geq 32 \dots \dots \dots (183)$$

Constraint of the lower limit of the methoyonine amino acid percentage in the finisher mash

$$\bullet 0.37C1+ 0.37C2+ 0.4C3+ 0.35C4+ 0.3C5+ 0.42C6+ 0.47C7+ 1.91C8+ 2.79C9+ 1.33C10+ 1.47C11+ 2.22C12+ 1.86C13+ 2.32C14+ 0.29C15+ 0.47C16+ 0.48C17+ 0.59C18+ 0.41C19+ 0.51C20+ 0.55C21 \leq 75 \dots \dots \dots (184)$$

Constraint of the lower limit of the thiamin amount in the finisher mash

$$\bullet 4.4C1+ 5.1C2+ 4C3+ 3.9C4+ 0.6C5+ 7.9C6+ 2.6C7+ 0.2C8+ 0.2C9+ 4C10+ 6.6C11+ 2.86C13+ 0.2C14+ 3.6C15+ 4C20+ 0.9C21 \geq 160 \dots \dots \dots (185)$$

Constraint of the lower limit of thiamin amount in the finisher mash

$$\bullet 4.4C1+ 5.1C2+ 4C3+ 3.9C4+ 0.6C5+ 7.9C6+ 2.6C7+ 0.2C8+ 0.2C9+ 4C10+ 6.6C11+ 2.86C13+ 0.2C14+ 3.6C15+ 4C20+ 0.9C21 \leq 270 \dots \dots \dots (186)$$

Constraint of the upper limit of the riboflavin amount in the finisher mash

$$\bullet 1.2C1+ 2C2+ 1.1C3+ 1.2C4+ 0.6C5+ 3.1C6+ 0.9C7+ 1.5C8+ 2.2C9+ 2.6C10+ 3.3C11+ 3.3C12+ 3.74C13+ 4.6C14+ 18.9C15+ 44.4C19+ 15.4C20+ 2.4C21 \geq 360 \dots \dots \dots (187)$$

Constraint of the upper limit of the riboflavin amount in the finisher mash

$$\bullet 1.2C1+ 2C2+ 1.1C3+ 1.2C4+ 0.6C5+ 3.1C6+ 0.9C7+ 1.5C8+ 2.2C9+ 2.6C10+ 3.3C11+ 3.3C12+ 3.74C13+ 4.6C14+ 18.9C15+ 44.4C19+ 15.4C20+ 2.4C21 \leq 370 \dots \dots \dots (188)$$

Constraint of the lower limit of the pantothenic amount in the finisher mash

$$\bullet 12.1C1+ 6.5C2+ 5C3+ 11.4C4+ 3.3C5+ 29C6+ 13C7+ 10.3C8+ 2.9C9+ 15.2C10+ 14.5C11+ 10.1C12+ 6.38C13+ 3.3C14+ 33.9C15+ 82.9C19+ 32.8C20+ 39C21 \geq 860 \dots \dots \dots (189)$$

Constraint of the upper limit of the pantothenic amount in the finisher mash

$$\bullet 12.1C1+ 6.5C2+ 5C3+ 11.4C4+ 3.3C5+ 29C6+ 13C7+ 10.3C8+ 2.9C9+ 15.2C10+ 14.5C11+ 10.1C12+ 6.38C13+ 3.3C14+ 33.9C15+ 82.9C19+ 32.8C20+ 39C21 \leq 1000 \dots \dots \dots (190)$$

Constraint of the lower limit of the niacin amount in the finisher mash



$$\bullet 56.6C1+ 57.4C2+ 22.9C3+ 42.7C4+ 14.1C5+ 209C6+ 20C7+ 49.9C8+ 5.5C9+ 30.4C10+ 26.8C11+ 220C12+ 30.8C13+ 47.1C14+ 10.6C15+ 500C19+ 54.6C20+ 46C21 \geq 2270 \dots (191)$$

Constraint of the upper limit of the niacin amount in the finisher mash

$$\bullet 56.6C1+ 57.4C2+ 22.9C3+ 42.7C4+ 14.1C5+ 209C6+ 20C7+ 49.9C8+ 5.5C9+ 30.4C10+ 26.8C11+ 220C12+ 30.8C13+ 47.1C14+ 10.6C15+ 500C19+ 54.6C20+ 46C21 \leq 2800 \dots (192)$$

Constraint of vegetable protein sources in the finisher mash

$$\bullet C8+ C9+ C10+ C12+ C13+ C16+ C17+ C18+ C19+ C20+ C21 \leq 35 \dots (193)$$

4. The solution of the three mathematical models

The software winqsb was used to find the optimal solution for the three mathematical models

Table 6. The optimal values of the decision variables for the three mashes

	Decision variable	Optimal value	Decision variable	Optimal value	Decision variable	Optimal value
1	A1	0	B1	0	C1	10.3174
2	A2	14.4707	B2	0	C2	0
3	A3	0	B3	11.8519	C3	38.7385
4	A4	10	B4	10	C4	0
5	A5	19.7253	B5	23.0473	C5	0
6	A6	0.1427	B6	0	C6	0
7	A7	5	B7	5	C7	0.3822
8	A8	5	B8	5	C8	4.5824
9	A9	1.0967	B9	5	C9	5
10	A10	29.5640	B10	17.6989	C10	0.9694
11	A11	0	B11	0	C11	5.8324
12	A12	3.7959	B12	0	C12	0
13	A13	0	B13	0	C13	0
14	A14	0	B14	0	C14	0.7385
15	A15	1.6319	B15	1.4845	C15	0.2043
16	A16	3.5992	B16	10	C16	10
17	A17	0	B17	0	C17	0
18	A18	0	B18	2.1753	C18	10
19	A19	0.7237	B19	3.5841	C19	0
20	A20	0	B20	0	C20	2.8169
21	A21	0	B21	0.0824	C21	5.9933

22	A22	1	B22	0.8256	C22	0.1747
23	A23	4	B23	4	C23	0
24	A24	0	B24	0	C24	4
25	A25	0	B25	0	C25	0
26	A26	0	B26	0	C26	0
27	A27	0.25	B27	0.25	C27	0.25

Table 7. The characteristic of the three mashes

		Initial mash	Grower mash	Finisher mash
1	Protein%	24	22	20
2	Energy	2800	3000	3200
3	Lipids%	6.5	6.15	6.07
4	Humidity%	9.9	9.7	9.59
5	Crude fibers%	5.27	4.6	4
6	The ash%	5	4.19	3
7	The phosphor(mg)/kgm	0.4	0.35	0.19
8	The calcium(mg)/kgm	1.05	0.916	0.38
9	The sodium(mg)/kgm	0.241	0.22	0.19
10	Lysine(mg)/kgm	1.23	1.02	0.834
11	Methionine(mg)/kgm	0.8	0.724	0.69
12	Thiamin(mg)/kgm	2.64	1.92	2.63
13	Riboflavin(mg)/kgm	2.2	2.95	1.6
14	Pantothenic(mg)/kgm	10	10	8.2
15	Niacin(mg)/kgm	40	37.5	24
16	Energy/protein ratio	116.6	136.6	160
17	Calcium/phosphor ratio	2.5	2.57	2
18	Cost of 100kgm of diet(dinar)	57210.2	56519.53	68159.22

Table 8. Sensitivity analysis of the objective function coefficients for the initial mash

	Decision variable	Solution value	Reduced cost	Unit cost or profit C(j)	Allowable Min. C(j)	Allowable Max. C(j)
1	A1	0	48.6237	510	461.3763	M
2	A2	14.4707	0	430	419.6268	432.2101
3	A3	0	55.4077	490	434.5923	M
4	A4	10	0	410	-M	445.4439
5	A5	19.7253	0	290	282.0684	300.1610
6	A6	0.1427	0	400	395.8073	411.8904

7	A7	5	0	400	-M	570.3338
8	A8	5	0	958	-M	1063.978
9	A9	1.0967	0	1450	1444.2870	1480.285
10	A10	29.5640	0	838	750.9686	840.4678
11	A11	0	20.0040	910	889.9960	M
12	A12	3.7959	0	960	947.4886	985.7807
13	A13	0	2026.921	3000	973.0792	M
14	A14	0	539.2134	1750	1210.7870	M
15	A15	1.6319	0	850	442.8283	3553.218
16	A16	3.5992	0	510	507.0192	580.3093
17	A17	0	565.1914	1210	644.8086	M
18	A18	0	55.0256	710	654.9744	M
19	A19	0.7237	0	710	657.1424	1199.236
20	A20	0	161.3866	510	348.6134	M
21	A21	0	313.9939	610	296.0061	M
22	A22	1	0	110	-M	373.8867
23	A23	4	0	110	-M	235.3386
24	A24	0	1602.006	1800	197.9938	M
25	A25	0	1247.240	1930	682.7601	M
26	A26	0	1067.241	1750	682.7590	M
27	A27	0.25	0	100	-M	M

Table 9. Sensitivity analysis of the objective function coefficients for the grower mash

	Decision variable	Solution value	Reduced cost	Unit cost or profit C(j)	Allowable Min. C(j)	Allowable Max. C(j)
1	B1	0	35.0649	510	474.9351	M
2	B2	0	16.9957	430	413.0043	M
3	B3	11.8519	0	490	481.7990	497.8255
4	B4	10	0	410	-M	562.0387
5	B5	23.0473	0	290	276.4873	300.2552
6	B6	0	465.7074	400	-65.7074	M
7	B7	5	0	400	-M	531.8535
8	B8	5	0	958	-M	986.4819
9	B9	5	0	1450	-M	1483.937
10	B10	17.6989	0	838	821.0748	862.1385
11	B11	0	146.2615	910	763.7385	M
12	B12	0	32.0064	960	927.9937	M
13	B13	0	2342.755	3000	657.2448	M
14	B14	0	41.6486	1750	1708.3510	M
15	B15	1.4845	0	850	-84.9726	1085.859

16	B16	10	0	510	-M	659.6033
17	B17	0	459.1376	1210	750.8624	M
18	B18	2.1753	0	710	699.1809	720.4007
19	B19	3.5841	0	710	502.3945	761.1245
20	B20	0	32.1007	510	477.8993	M
21	B21	0.0824	0	610	568.7891	646.5701
22	B22	0.8256	0	110	-874.6069	165.3118
23	B23	4	0	110	-M	487.8731
24	B24	0	1603.782	1800	196.2182	M
25	B25	0	789.8797	1930	1140.120	M
26	B26	0	609.8797	1750	1140.120	M
27	B27	0.25	0	100	-M	M

Table 10. Sensitivity analysis of the objective function coefficients for the finisher mash

	Decision variable	Solution value	Reduced cost	Unit cost or profit C(j)	Allowable Min. C(j)	Allowable Max. C(j)
1	C1	10.3174	0	510	500.6506	524.6102
2	C2	0	23.9956	430	-M	M
3	C3	38.7385	0	490	475.5099	503.5728
4	C4	0	-158.3258	410	-M	M
5	C5	0	-18.2852	290	-M	M
6	C6	0	112.6969	400	-M	M
7	C7	0.3822	0	400	-57.5590	412.0724
8	C8	4.5824	0	958	822.6099	1537.564
9	C9	5	0	1450	-M	M
10	C10	0.9694	0	838	819.5104	908.7286
11	C11	5.8324	0	910	789.6345	933.5669
12	C12	0	-484.8367	960	-M	M
13	C13	0	2374.8970	3000	-M	M
14	C14	0.7385	0	1750	1402.0960	1884.619
15	C15	0.2043	0	850	102.3680	2717.005
16	C16	10	0	510	-M	545.3395
17	C17	0	0	1210	-M	M
18	C18	10	0	710	-M	M
19	C19	0	-1857.332	710	-M	M
20	C20	2.8169	0	510	305.7042	766.9684
21	C21	5.9933	0	610	-111.7691	825.5251
22	C22	0.1747	0	110	80.4122	33254.94
23	C23	0	0	110	-M	M
24	C24	4	0	1800	-M	M

25	C25	0	971.7833	1930	-M	M
26	C26	0	791.7833	1750	-M	M
27	C27	0.25	0	100	-M	M

5. Conclusions

1. The three mashes mentioned their specifications in table 7 correspond to the specifications determined by the nutritionists and they are guaranteed of quality while the ready-made-feed that is purchased from the local markets is not guaranteed of quality.

2. The cost of 100 kgm of the initial mash is extracted from the application of the linear programming method 57210.2 iraqi dinar, while the cost of buying 100 kgm from the local markets is 70000 iraqi dinars, meaning that the price of 100 kgm of the developing diet resulting from the application of the linear programming method is reduced by 12789.78 iraqi dinars from the prices in the local markets

3. The cost of 100 kgm of the grower mash is extracted from the application of the linear programming method 56519.53 iraqi dinar, while the cost of buying 100 kgm from the local markets is 70000 iraqi dinars, meaning that the price of 100 kgm of the developing diet resulting from the application of the linear programming method is reduced by 13480.47 iraqi dinars from the prices in the local markets

4. The cost of 100 kgm of the finisher mash is extracted from the application of the linear programming method 68159.22 iraqi dinar, while the cost of buying 100 kgm from the local markets is 70000 iraqi dinars, meaning that the price of 100 kgm of the developing diet resulting from the application of the linear programming method is reduced by 1840.78 iraqi dinars from the prices in the local markets

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